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CONTINUED ROOTS.

BY T. S. E. DIXON, CHICAGO, ILLINOIS.

THE expression $\sqrt{q-p\sqrt{[q-p\sqrt{(q--\&c.)}]}}$, from its analogy to the continued fraction

$$\cfrac{q}{p+\cfrac{q}{p+\cfrac{q}{p+\&c.,}}}$$

would be called a *Continued Root.* The two expressions are equivalents, each being an expression for the roots of the quadratic equation $x^2+px=q$.

Continued roots, however, seem to have a wider application than continued fractions, since they may be formed so as to express not only the roots of a quadratic equation, but also the roots of an equation of any degree of the form $x^n\pm px=\pm q$.

Thus $\sqrt[3]{q+p\sqrt[3]{[q+p\sqrt[3]{(q+\&c.)}]}}=$ the roots of the cubic equation $x^3-px=q$, $\sqrt[n]{q+p\sqrt[n]{[q+p\sqrt[n]{(q+\&c.)}]}}=$ the roots of the equation $x^n-px=q$.

A combination of the two expressions may also be used to represent the roots of these equations, affording radicals of one less degree. Thus in the equation $x^3-px=q$,

$$x=\sqrt{p+\cfrac{q}{\sqrt{p+\cfrac{q}{\sqrt{p+\&c.}}}}}$$

The roots of any trinomial equation, of whatever degree, may be expresseed in terms of the known quantities under the form of a continued root. Thus if $x^m+px^n=q$, $x=\sqrt[m]{q-p.\sqrt[m\div n]{[q-p.\sqrt[m\div n]{(q-\&c.)}]}}$.

Again, if $x^{m+n}+px^n=q$,

$$x=\sqrt[n]{\cfrac{q}{p+\sqrt[n\div m]{\cfrac{q}{p+\sqrt[n\div m]{\cfrac{q}{p+\&c.}}}}}}$$

Methods analogous to those which give approximate values to continued fractions may also be applied to continued roots. These roots also give a method for the solution of biquadratic equations.

If $x^2-px=q$ (1), $x=\sqrt{q+p\sqrt{[q+p\sqrt{(q+\&c.)}]}}$, $x^2=q+p\sqrt{q+p\sqrt{(q+\&c.)}}$, $x^4=q^2+p^2q+(2pq+p^3)\sqrt{q+p\sqrt{(q+\&c.)}}$, whence $x^4-(2pq+p^3)x=q^2+p^2q$ (2). Given $x^4-Ax=B$. Then $2pq+p^3=A$, $q^2+p^2q=B$, whence the cubic $p^6+4Bp^2=A^2$. Obtaining now the values of p and q, we have from (1), $x=\frac{1}{2}[p\pm\sqrt{(p^2+4q)}]$.

If the biquadratic is of the general form $x^4+Ax^2+Bx=C$, add to eqn. (2) $rx^2=qr+prx$, transpose and $x^4+rx^2-(p^3+2pq+r)x=q^2+p^2q+qr$, whence the cubic $p^6+2Ap^4+(A^2+4C)p^2=B^2$; where x is found as above.

It is interesting to note that continued roots have their converse, which by analogy would be called *Continued Powers.* Thus the four expressions following are equivalents, each being an expression for the roots of the equation $x^2+px=q$.

$$\frac{q}{p+\frac{q}{p+\&c.}},\qquad -p+\frac{q}{-p+\frac{q}{-p+\&c.}},\qquad \sqrt{q-p\sqrt{(q-\&c.)}},\quad \frac{q}{p}-\frac{1}{p}^2\left(\frac{q}{p}-\frac{1}{p}^2\left(\frac{q}{p}\&c.\right.\right.$$

LEAST SQUARES.

BY R. J. ADCOCK. (CONTINUED FROM P. 184, VOL. IV.)

LIMITS OF ERRORS.—By first and second formulas on pages 183 and 184,

$$p=\frac{n\,{}^{*}}{4\pi m\,S(d_1^2)^{\dagger}}\tag{1}$$

is the probability that any point of the total number $4\pi S(d_1^2)m$, shall be at or on any required point, line or surface, and

$$p=\left(\frac{n}{4\pi m\,S(d_1^2)}\right)^n\tag{2}$$

is the probability for n points of the same number. Each of which values of p is greatest when $S(d_1^2)=$ a minimum.

Let it be assumed that, in different sets of n points similarly observed, the minimum $S(d_1^2)$ remains unchanged, or that the total number of points, in determining the probability in (1) or (2), remains the same, which is nearly true in practice and more nearly so as n is increased.

Now the number of points which are at the distance l from any point is $4\pi ml^2$, therefore, under the assumption, the probability that a random point shall be at the distance l from the foot of any one of the normals $d_1, d_2 \ldots d_n$, is

$$p=\frac{4\pi mnl^2}{4\pi m\,S(d_1^2)}=\frac{nl^2}{S(d_1^2)}\ldots(3)\quad \text{Hence } l=\sqrt{\left(\frac{pS(d_1^2)}{n}\right)}\tag{4}$$

is the limit within which equal normals or errors are situated when their probability is p. $p=1$, gives $l=\surd[S(d_1^2)\div n]\ldots(5)=$ what is called the mean error, that is, if each error is of the same magnitude it will be $\surd[S(d_1^2)\div n]$. And $p=\frac{1}{2}$, gives $l\,\surd[S(d_1^2)\div 2n]\ldots(6)$ which is called the probable error of a single observation, that is, if $\frac{1}{2}$ the errors are equal, each will be $\surd[S(d_1^2)\div 2n]$. Hence this value should include one half of the n errors, and a single one is as likely to be less than, as to exceed it.

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*Instead of 1 as the numerator in the formula near the bottom of p. 183, Vol. IV, read n. Also, for π in the same formula and in the two formulas on p. 184, read 4π.

†$S(d_1^2)=d_1^2+d_2^2+d_3^2+\ldots+d_n^2$.